Relationship between λ_i, $h(t)$ and $h[n]$

Recall:

- If we send

$$s(t) = \underset{t\in[-T_L,0]}{[\text{cyclic extension}]} + \underbrace{\frac{1}{N}\sum_{i=0}^{N-1} A_i e^{j2\pi f_i t}}_{x(t) \text{ for } t\in[0,T]}$$

Then, for $t \in [0,T]$, we receive

$$r(t) = \frac{1}{N}\sum_{i=0}^{N-1} \lambda_i A_i e^{j2\pi f_i t} + \underline{\text{noise}}$$

where $\lambda_i = h_F(f_i)$

- We choose $f_i = \frac{i}{T}$. Then, the samples of $x(t)$ are

$$\underline{x} = DFT^{-1}(\underline{A}) \quad , \quad \underline{A} = (A_0, A_1, \dots A_{N-1})$$

To the interpolation filter we send $(\underset{\uparrow}{\tilde{\underline{x}}}, \underline{x})$ cyclic prefix

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Question: Is the output of the interpolation filter going to be $s(t)$.

Actually not.

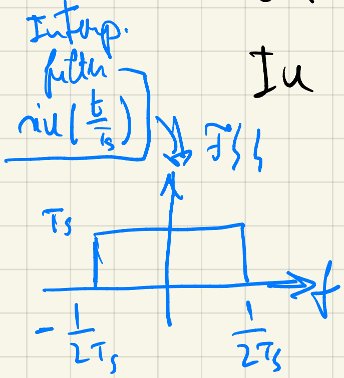
The samples of $e^{j2\pi f_i T_s}$ are the same as the samples of $e^{j2\pi\left(f + \frac{1}{T_s}\right) i T_s}$ $\quad i=0, \dots N-1$

More generally, the samples of $e^{j2\pi ft}$ are identical to the samples of $e^{j2\pi\left(f+\frac{k}{T_s}\right)t}$ for any integer k. (We sample at iT_s, $i=0,\dots M-1$).

If we send these samples to the interpolation filter, is the output going to be $e^{j2\pi ft}$ or $e^{j2\pi\left(f+\frac{k}{T_s}\right)t}$?

<u>It depends</u>:

It outputs the one and only one for which the frequency $\left(f+\frac{k}{T_s}\right)$ lies in the passband of the interpolation filter.



In our case, the interpolation filter has the passband in $\left[-\frac{1}{2T_s}, \frac{1}{2T_s}\right]$ or equivalently

$$\left[-\frac{M/2}{T}, \frac{M/2}{T}\right] \qquad (T = MT_s)$$

Thus, s_i is carried by the complex exponential of frequency

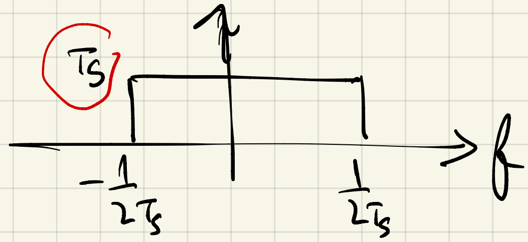
$$f_i = \begin{cases} \frac{i}{T}, & i=0,\dots M/2-1 \\ \frac{i-M}{T}, & i=\frac{M}{2},\dots M-1 \end{cases}$$

$$s_i = \begin{cases} h_F\left(\frac{i}{T}\right), & i=0,\dots \frac{M}{2}-1 \\ h_F\left(\frac{i-M}{T}\right), & i=\frac{M}{2},\dots M-1 \end{cases}$$

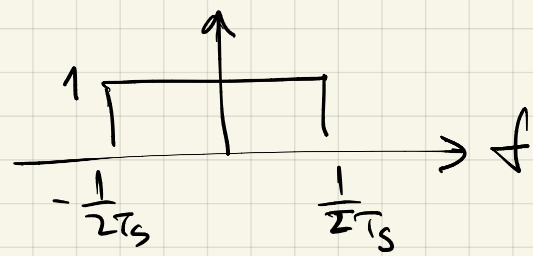
$\lambda_i = T_s * DFT(\underline{h})$ where $\underline{h} = [h(0), h(T_s), \ldots h((N-1)T_s)]$

We have assumed that the interpolation filter $p(t)$ and that the lowpass filter $q(t)$ are ideal

$$p(t) = \mathrm{sinc}\left(\frac{t}{T_s}\right) \xleftrightarrow{\mathcal{F}\{\}}$$



$$q(t) = \frac{1}{T_s}\,\mathrm{sinc}\left(\frac{t}{T_s}\right) \xleftrightarrow{\mathcal{F}\{\}}$$



In practice, $p(t)$ is at best a truncated and delayed sinc (delayed for causality). The same applies to $q(t)$.

More generally, suppose $p_F(f)$ and $q_F(f)$ are arbitrary frequency responses with support $\left[-\frac{1}{2T_s}, \frac{1}{2T_s}\right]$.

Then, the eigenfunction at frequency f_i is scaled by $\frac{p_F(f_i)}{T_s} h_F(f_i)\, q_F(f_i)$

for remove the T_s from $p(t)$

$$= \frac{1}{T_s} h_{o,F}(f_i)$$

with $h_o = p * h * q$ (o for overall)

Hence $\lambda_i = \frac{1}{T_s} h_{o,F}(f_i)$

Recall OFDM

$$e^{j2\pi f_i t} \longrightarrow \boxed{h(t)} \longrightarrow \lambda_i e^{j2\pi f_i t}$$

where $\underline{\lambda_i = h_F(f_i)}$

If we sample $h_0(t)$, we obtain the sample-level impulse response $h_0[i]$, $i = 0 \dots L-1$

Let $\underline{h_0} = (h_0[0], h_0[1], \dots h_0[L-1])$

Recall that the i-th component of $DFT(\underline{h_0})$ is $\frac{1}{T_s} h_{0,F}(f_i)$ (first lecture)

Hence $\underline{\lambda_i = DFT(\underline{h_0})}$